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SELF-ALIGNING ROCKETS

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A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

BALLISTIC RESEARCH LABORATORIES

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ABSTRACT

It is suggested that the head and the motor of a fin-stabilized rocket be joined not rigidly, but by a trapezoidal linkage, with the larger parallel side of the linkage located forward and joined to the motor, so that the linkage is under tension and is stable with respect to the angle between the head and the motor. The apparently inavoidable linear misalignment of the jet thrust then can be converted, in effect, into the relatively harmless angular misalignment. The linkage possesses a number of other useful features, and it does not seem impossible that it may lead to a radical increase in the accuracy of rockets or boosters.

1. INTRODUCTION

It is understood, tentatively, that one of the basic - and apparently inavoidable - reasons for the dispersion of non-spinning fin-stabilized rockets is the accidental misalignment of the jet thrust. This may be the angular misalignment (thrust at an angle to the axis of the rocket, or there is present a cross-force at the center of mass of the rocket), or the linear misalignment (thrust offset from the axis of the rocket, or there is present a torque whose vector is perpendicular to the axis of the rocket). It is furthermore understood that of these two the linear misalignment is the more harmful one, especially when the burning distance is great; and that it is principally for this reason that efforts are being made to reduce the burning distance, sacrificing thereby certain inherent advantages of rocket in comparison with a recoilless gun (1,2).

The object of this paper is to record the possibility of connecting the motor and the head of a rocket by a linkage such that the motor would have a modicum of freedom of rotation with respect to the head, so that in the presence of an accidental linear misalignment it will simply turn through a certain angle, converting the harmful linear misalignment into the relatively harmless angular misalignment. Only the bare outlines of the principles of such a linkage are presented, and the subject is treated more as an abstract problem in Mechanics rather than an Ordnance design.

The suggestion hinges on certain interesting features of the common trapezoidal linkage, to which we now turn our attention.

2. TRAPEZOIDAL LINKAGE

Consider a symmetrical trapezoidal linkage (Fig. 1), whose small parallel side (located at the bottom, and connected rigidly to the head of the rocket, which is located above) for the moment may be imagined stationary. The large parallel side (located on top, and connected rigidly to the motor, which is located below) is moving, and has an external force (the symmetrical jet thrust) applied upwards along its perpendicular bisector (the axis of the motor). The non-parallel sides (say, "arms") are under pure tension. We wish to inspect the forces and the torques exerted by the linkage on these two bodies.

Let the height of the trapezoid be h, and let the ratio of the sides be $1 + 1/Q$. The center of rotation of the motor is always at the point of intersection of the arms (3); and in particular, for the undistorted trapezoid it is on the axis of the head at a distance hQ from the small side (4). As the motor rotates, the center of rotation moves sidewise, (5,6,7) but a point on the motor that was in that center for the undistorted trapezoid, does not move with the center of rotation; rather, the motion of the motor is rolling, and that point acquires only a downward velocity, which - it is easy to see - is of the second order

(1) Numbers in parentheses refer to the remarks in the Appendix.

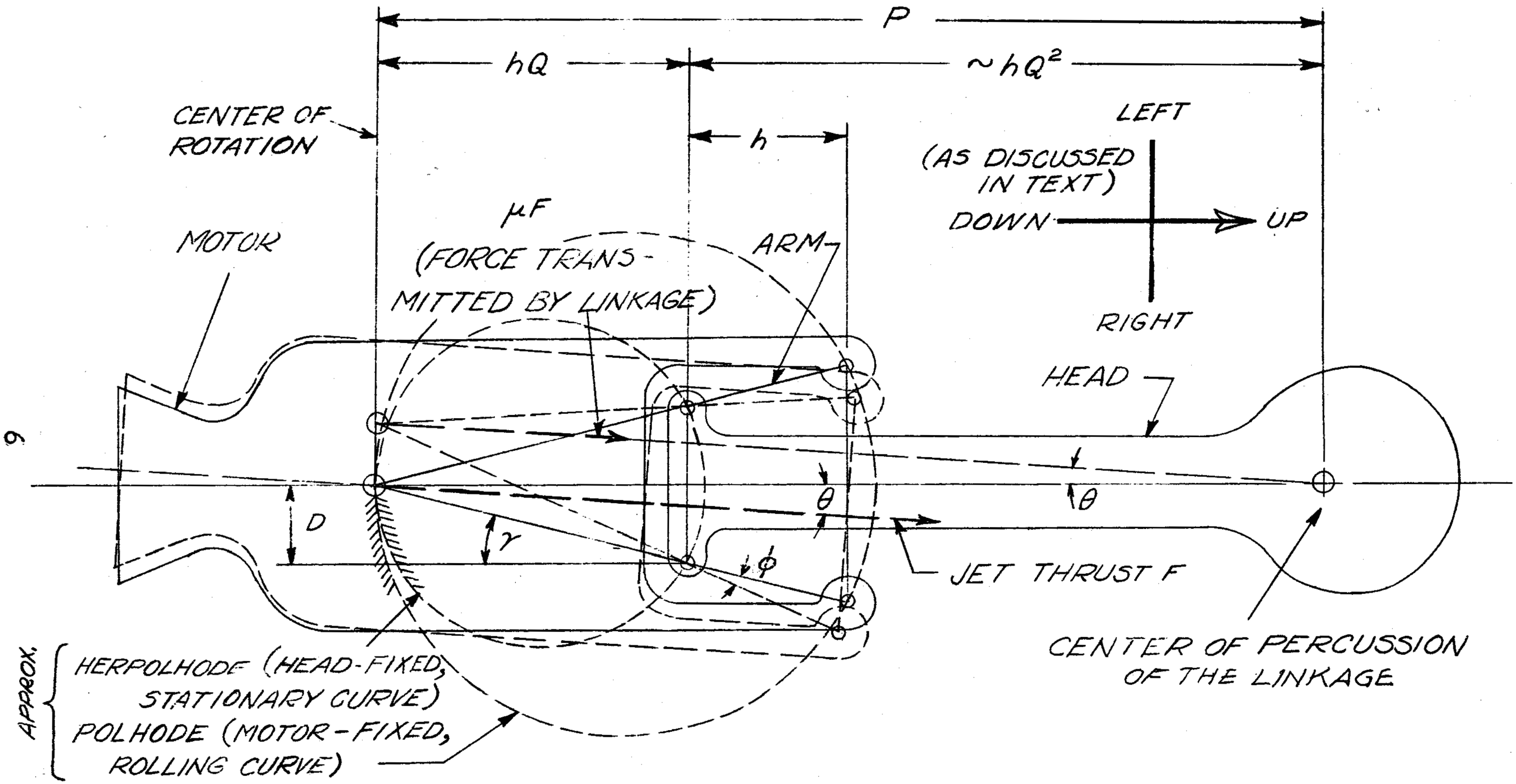

FIG. 1 - TRAPEZOIDAL LINKAGE

with respect to the angle through which the motor turns (8). Thus to the first order we might say that the motor simply turns about a point fixed (approximately) on the axis of the head. Obviously, the original reason for this linkage is to introduce this effective pivot somewhere near the center of mass of the motor, far away from compact juncture of the motor and the head.

The resultant of the forces exerted by the two arms, however, passes through the intersection of these arms, (9) viz., through a point generally removed from the axis of the motor. It is obvious that this resultant will be - at least approximately - parallel to the applied force; and that there will therefore be a restoring torque, tending to turn the motor back into the symmetrical position. This inherent stability of this linkage amounts to the fact that the two arms act as a couple of stretched and linked pendulums.

More interesting is the fact that this resultant force will indeed be parallel to the applied force: this is the result of our assumptions of small angles and of the absence of applied forces other than the symmetrical jet thrust (10). Since this resultant passes through a point whose distance from the axis of the head is proportional to the angle between this resultant and the axis (11), the important consequence is that this resultant always passes through a point which is fixed - to the first order - in the axis of the head. It can be easily shown that this point is located at the distance of approximately hQ^2 from the small side of the trapezoid (12). We shall call this point (in the writer's ignorance of other established name for it) the center of percussion of the trapezoidal linkage (13). Obviously, it is approximately at this point that the center of mass of the head should be located: for then the linkage will exert no torque upon the head.

The linkage is thus characterized by two purely geometrical centers (those of rotation and percussion), separated by the distance of approximately $P = h(Q + Q^2)$, and the parameters h and Q may be readily selected to suit the other features of the design. We may remark at this point (anticipating our rudimentary discussion of the performance of such rockets) that it might be more practical to accept a center of percussion far ahead of the center of mass of the head. The parameter Q is readily recognized as roughly the ratio of the distance from the small side to the center of percussion to the distance from the center of rotation to the small side. To make the linkage compact we need small h and large Q; and an insistence on the coincidence of center of mass of the head with the center of percussion would limit the application of this linkage to very-low-velocity rockets.

We shall now attempt to discuss the expected performance of such rockets, commencing with a series of drastic simplifying assumptions (including the assumption of the center of mass of the head at the center of percussion of the linkage).

3. PERFORMANCE IN VACUUM

Assume no aerodynamic forces and torques; no gravity; no initial yaw or yawing; zero-length launcher; a constant mass m of the rocket and a constant acceleration A. In the presence of the angular misalignment (angle α between the axis of the rocket and the assumed constant direction of thrust) the conventional rigid rocket simply takes off in the direction α, without rotation. In the presence of the linear misalignment (distance M between the line of thrust and the center of mass of the rocket), the conventional rigid rocket turns with an angular acceleration FM/mk^2, where F = thrust and k = radius of gyration; in time t the rocket turns through the angle $FMt^2/2mk^2$, so that there is a cross-force $F^2Mt^2/2mk^2$, a cross acceleration $F^2Mt^2/2m^2k^2$, and a cross-velocity $F^2Mt^3/6m^2k^2$; dividing this by the axial velocity Ft/m, we have the instantaneous deflection as $FMt^2/6mk^2$; and noting that $F/m = A$, and at the end of burning $At^2/2$ is the burning distance B, the final deflection is $MB/3k^2$. Thus a decrease in B is usually desirable, even though it results in a greater blast at the gunner, higher pressure in the motor, a heavier motor, lower velocity and a less efficient rocket. More specifically, if both M and α are due to an accidental turning of the nozzle through the angle α, we may assume that $M = -\alpha L/2$ approximately (L = length of the rocket), so that ratio of the deflection due to M to that due to α is $-LB/6k^2$; and if k^2 is roughly $L^2/12$, the ratio is $-2B/L$, i.e., much more than 1 in magnitude, as it was mentioned at the outset. Of the many refinements which modify such a conclusion, the most essential one is the introduction of the righting torque $K_M\rho d^3u^2\delta$ (where K_M = moment coefficient, ρ = density of the air, d = caliber, u = velocity and δ = angle of yaw); but even then the skeleton of the above reasoning survives, the phenomenon being merely limited to the first quarter-period of yaw, or so. The most essential characteristic of the motion then is the length of the period of yaw, rather than B; but the importance of B still survives, essentially because the length of the period of yaw is never small enough.

With our rocket the basic form of motion is the oscillation of the motor about the axis of the head. The force transmitted by the linkage is μF, where μ is the ratio of the mass of the head to the mass of the whole rocket; the restoring torque is $\mu FP\theta$, where θ is the angle of rotation of the motor. Since the coefficient μFP is obviously much greater than $K_M\rho d^3u^2$ can be conveniently made, and since the moment of inertia of the motor alone is obviously much less than the moment of inertia of the rigid rocket, the frequency of this oscillation must be comparatively high; the residual deflection (produced mainly during the first quarter-period, or so, of this oscillation) may be then, in the first approximation, neglected; while it should not be difficult to damp such rapid oscillations out rapidly. Then in the presence of a linear misalignment represented by a constant torque of the magnitude FM, the motor will simply settle down rapidly at the angle θ such that $FM = \mu FP\theta$, or $\theta = M/\mu P$; which will be the effective angular misalignment of the new configuration of our rocket, that may be now considered as rigid. Comparing the deflection $M/\mu P$ with the deflection $MB/3k^2$ of the same rocket that would have occurred in vacuum had the linkage been locked, we see that our linkage would reduce the deflection in the ratio $3k^2/B\mu P$; if $k^2 = L^2/12$, $\mu = 1/2$, and $P = L/2$, this ratio is L/B; i.e., the possible gain in accuracy seems to be impressive enough. It should be noted that this radical gain merely reflects the awkwardly large deflection of the non-spinning rocket in vacuum, and there is no intention to infer that a large B will be advantageous; it is merely that the gain will be more conspicuous with the more awkward rockets (having the larger B).

Let us now inspect qualitatively the results of rescinding the assumption that the center of percussion of the linkage is at the center of mass of the head (which was implied when we assumed no rotation of the head, and $P = L/2$). Specifically, let us leave the center of rotation and the center of mass of the motor coincident, but let us change the proportion of the linkage, by making P larger than before. Now, as the motor turns through some angle under the action of the misalignment torque FM, it begins to exert some torque on the head; and as the head turns, the restoring torque of the linkage causes the motor to follow the head, so that our rocket begins to exhibit the same type of behaviour as the rigid rockets do. It seems reasonable to suppose, however, that the torque transmitted to the head could be made much smaller, particularly in view of the fact that the angle $\theta = M/\mu P$ will be decreased; so that our mechanism might achieve a reduction, if not a complete suppression, of the deflection due to a linear misalignment.

More exact recission of this assumption (as well as the recission of the assumption that the center of rotation is at the center of mass of the motor) will be best handled in conjunction with the introduction of the aerodynamic torques and forces - which will also introduce that annoying complication, the cross-force that may be transmitted by this linkage.

4. EFFECT OF THE AERODYNAMIC FORCES

We thus see that a rocket which had originally a symmetric exterior contour but possessed a linear misalignment of the jet will be rapidly changed into a new, bent, configuration in which the jet thrust will pass through the center of mass of rocket. In this new configuration the rocket will possess an angular misalignment (and possibly, some residual linear misalignment, if the center of percussion of the linkage is in front of the center of mass of the head); therefore, in vacuo this rocket will still be deflected (even though by a much smaller amount) in the same direction as it would have been deflected if it were rigid. If the exterior surface of this rocket remained symmetrical, the deflection due to the residual linear misalignment would be diminished by the action of the righting torque, but the deflection due to the angular misalignment will be somewhat increased by this action. Let us now inspect briefly the effect of the distortion of the exterior surface of the rocket.

If we assume that in this new configuration the rocket is effectively a rigid body, we may recollect that a bent non-spinning fin-stabilized shell is usually assumed to fly, eventually, along its axis of zero moment; in which angular position the shell is acted upon by a steady lift, and therefore glides. The rocket will only tend to this position and it may be that it will never achieve it; yet, a brief consideration of this limiting case seems relevant. Since the righting moment acting upon the finned motor must then be balanced by the overturning moment acting upon the head, in this angular position both the motor and the head are turned in the direction opposite to that of the deflection of this rocket in vacuo (the head being turned through a greater angle). The deflection due to this steady lift, then, is in the direction opposite to the deflection in vacuo. It would seem therefore - in principle - that the rocket may be so proportioned (intentionally or accidentally) that the residual deflection is well-nigh zero, no matter what is the misalignment of the nozzle.

It is now interesting to observe that the fact that the rocket does not acquire this angular position at once (since the deformation is due mainly to the motion of the motor alone, rather than to such a motion of both motor and head as will leave the resultant aerodynamic moment as zero) happens to help further in compensating the deflection in vacuo. After the rocket is deformed, and until such time when it settles in the equilibrium yaw, there will act a restoring moment in a direction opposite to the deflection in vacuo; this will be larger than for the conventional rigid rocket, because the angle of attack of the fins is greater. Disregarding the subsequent oscillations of the assembly (viz., assuming them, for simplicity, to be rapidly damped out somehow) - it is easy to see that the effect of this moment upon the deflection will be that of a temporary linear misalignment in the opposite direction.

It is not clear at all, unfortunately, that the resultant assembly (elastic due to the presence of the restoring torque due to the jet thrust) may be assumed to act as a rigid body. In a more thorough study it might be necessary to study the vibrations of this assembly, as well as its oscillations, and to allow for the separation of the lift and righting torque into the components acting separately upon the motor and the head. The equations of the motion of the system can be written out easily enough, but they turn out to be entirely too complicated (and the evaluation of the aerodynamic components, too involved) for our present purposes. More fruitful at this time will be a mere qualitative inspection of the action of such a system.

We should now note that the linkage may transmit a cross force at the center of rotation; for, with respect to a force perpendicular to the axis of the trapezoid and passing through the center of rotation, the linkage acts simply as a triangular truss, or as a cantilever beam. Such a force may arise, for instance, as a result of exterior cross-force applied to the motor, or - as a reaction - as a result of an exterior turning moment applied to the head.

Consider first the lift applied to the motor alone, and consider it applied at the coincident centers of the rotation of the linkage, and of the mass of the motor (the moment of the lift, acting on the motor at its center of pressure, can be considered separately, as any other moment, such as the moment due to the asymmetry of the jet; similarly, we need consider here only a rocket with a symmetrical jet). The question is what effect does this lift produce on the head (viz., on the center-line of our rocket). If the conventional rocket had its center of pressure at this point (our center of rotation), the restoring moment due to lift would have to turn both the head and the motor; in our case it has only to turn the head, since it cannot turn the motor (the motor will be turned by the large torque due to the jet, as soon as the head turns with respect to the motor). Indeed, some of this lift will be absorbed in the linear acceleration of the head, and some, in the linear acceleration of the motor; but since it is so much easier to turn the head about its distant center of percussion* (15), it is obvious that only a small fraction of this lift will be absorbed in the linear acceleration of the head. Thus the linear acceleration of the motor will be larger than in the case of a solid rocket, and of the head - smaller. This means that the centerline of our rocket will be turning that much faster (16), as if the restoring torque were greater than it really is. At the first glance this may appear as pulling oneself up with the bootstraps; the fact of the matter is, the restoring torque has a smaller job to do, and therefore does it more effectively - and the job is finished later, by the introduction of an exterior large torque of the jet.

The reader may wish to consider next the effect of the lift acting upon the head alone (this lift is applied somewhere near the nose, and causes the overturning moment on the head). Since the effect of the cross-force is simple only when the cross-force is applied at the center of rotation, it may be easier to visualize the effect of such a lift if we transfer it to the center of rotation. The lift will then be accompanied by a very large overturning torque; but most of this torque will be cancelled by the restoring torque of the lift applied at the center of rotation. It remains therefore only to inspect the effect of the (overturning) moment applied to the head alone. At the first glance it might appear that this moment will be resisted only by the relatively small moment of inertia of the head alone. This is not so; a rotation of the head about its center of mass requires also a translational motion of the center of gravity of the motor; hence, the inertial resistance of the head will be much greater, i.e., the harmful effect of the overturning moment will be less strong than it might at first appear to be. Again, the exact behaviour of the system can be worked out by setting up the equations of motion; but the purpose of this presentation is merely to point out that the proposed system does not seem to possess any glaring shortcomings in comparison with a conventional rocket.

* Not the center of the percussion of the linkage!

Further annoying complications will be introduced into the theory if the designer desires to rescind the assumption of the coincidence of the centers of rotation and of the mass of the motor. The motor then will be no longer relatively free to turn about its center of mass; the rotation of the motor about its center of mass will then involve some sidewise translation of the center of rotation, and consequently, some cross-force at this center of rotation, as the inertial reaction of the head. In particular, if the center of rotation of the linkage is back of the center of mass of the motor, the rotation of the head will be in the same direction as the rotation of the motor. We might say that some of the moment applied to the motor will then be transmitted to the head. The effect will be similar to that of introducing a spring restraint between the motor and the head. In fact, when the center of rotation is made to recede into infinity - viz., when the trapezoidal linkage becomes a parallelogrammic linkage - the relative angular motion of the head and the motor becomes impossible, just as with the increase of the stiffness of the spring restraint our rocket eventually becomes the conventional rigid rocket.

The complication of the theory which results from the recission of the coincidence of those two centers may not be absolutely necessary; but it is interesting because it may further help in compensating the residual deflection, as well as in designing a more compact linkage. The cross-force transmitted by the linkage enters the equations of the motion of the system as an additional variable, associated with what is essentially an equation of restraint - which states that the center of rotation is fixed both in the motor and in the head. Unfortunately, even in the simplest case - an immovable head - this cross-force turns out to be a fairly clumsy function of the applied forces and torques, distances and moments of inertia, etc. Lagrangian methods, by virtue of their inherent elimination of the equations of restraints, should be helpful in this connection.

5. PROSPECTS

A crude sketch showing a possible practical method of manufacturing such linkage for artillery rockets is given in Fig. 2. The linkage is a compact, light and self-contained sub-assembly of the complete round, to which first the head, and finally the motor, are assembled, perhaps by the simple press fit; the forces transmitted by the surfaces of contact are essentially those of compression, so that no particularly heavy structure (such as threads) is necessary. The linkage takes the place of that joint which is ordinarily an awkward and inefficient part of the design of a rocket; it may be noted that the thermal insulation of the head from the motor is achieved automatically, but a separate and efficient closure of the front end of the motor is necessary.

The linkage consists of two discs clamped tightly together by a wire wrapped essentially in a zigzag manner. The surface of the contact between the two discs, heavily greased, is a spherical surface with the center at the center of rotation of the trapezoids. The zigzag, ideally, is rather like a square (more exactly, trapezoidal) wave, with the wires

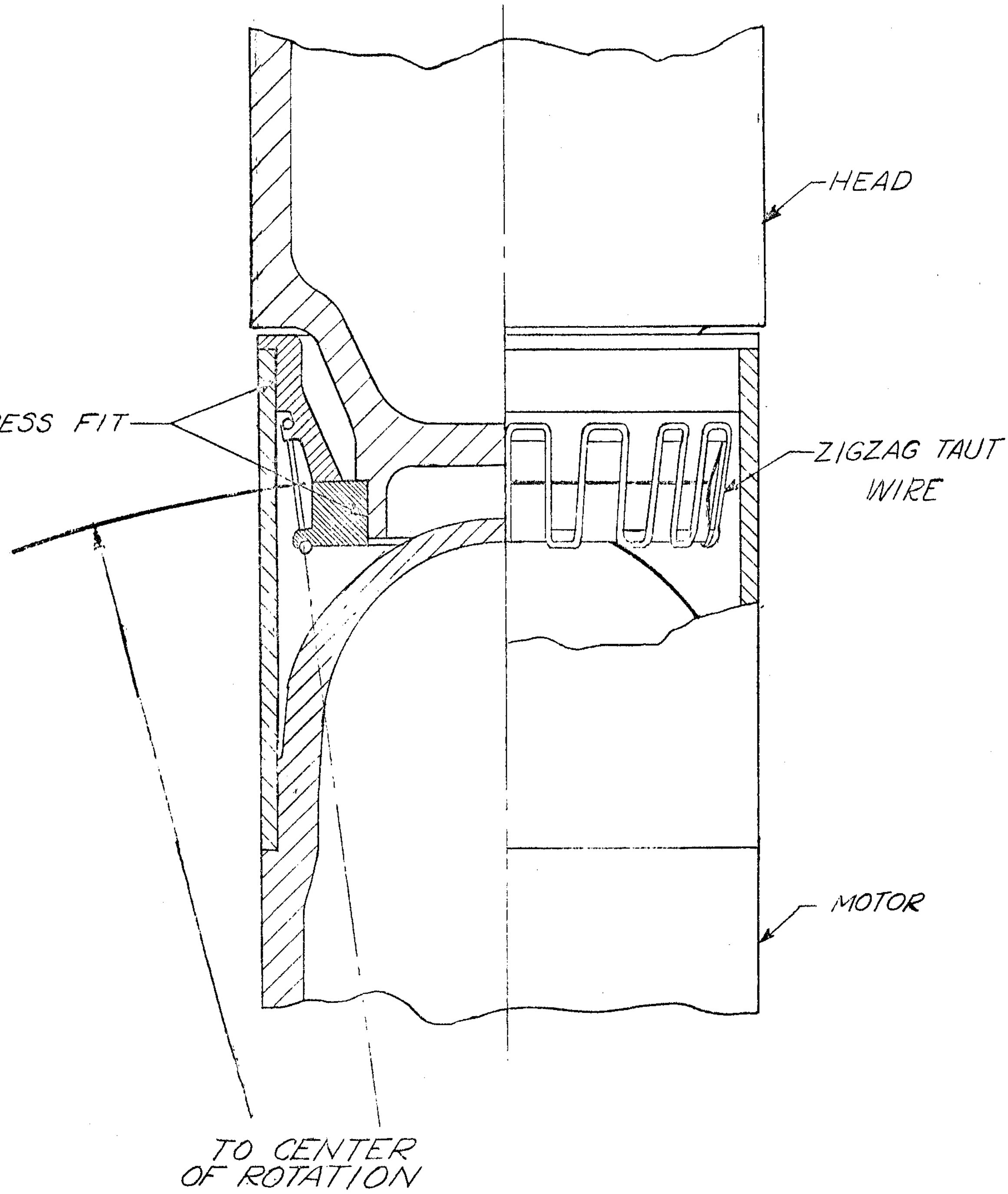

FIG. 2.-CRUDE SKETCH OF A POSSIBLE CONSTRUCTION OF THE LINKAGE

lying along the generatrices of the cone having its apex at the center of the spherical surface; then any two wires form a trapezoid with the same center of rotation. However, a minor departure from this condition may be beneficial, for it will merely increase the elastic stiffness of the rocket assembly, and will enable the linkage to transmit more twisting torque. The linkage can be readily made so that the surface of the contact between the two discs, in addition to being a spherical surface, includes some centering device; then in the absence of thrust between the motor and the head - viz., in handling, or in free flight - the rocket will be effectively a rigid body. This feature, however, is not essential, and in its absence the rocket will be simply a stiff elastic straight body. On the rise of the powder pressure in the motor the reaction of the head further stretches the wires so that the spherical surfaces separate (and the centering surfaces, if any, become unlocked). The grease (or other analogous device) serves to increase the damping, and yet to allow the resumption of the proper centering when the powder pressure subsides - so that in the free flight the rocket, in spite of the possible asymmetry of its jet during the burning, is again a symmetrical body.

The development of such rockets will be fraught with many auxiliary problems that are outside the province of this writer; they will require a cooperation of competent specialists and an energetic manufacturing experimentation. Thus, e.g., the rocket will certainly function even in the presence of "small" spin; but how small is small, and what are the effects of spin on this system, are questions outside the scope of this paper. Some problems (cf BRL 855D) loom larger in prospect than in the reality; some will not be realized until brought up by the experiments.

An analytical solution of the equations of motion, with the object of determining the parameters of the design so as to minimize the deflection, would be very desirable indeed; unfortunately, it seems to be extremely difficult. An obvious (and perhaps fashionable) path would be to assume a series of reasonable configuration of parameters, and to make the final selection on the basis of inspection of a large number of the solutions of the equations procured on the modern high-speed computing machines. In the opinion of the writer more fruitful will be an immediate extensive experimentation, with the designs based on the qualitative understanding of this mechanism - which this paper attempted to convey.

Serge J. Zaroodny
SERGE J. ZAROODNY

APPENDIX

REMARKS

The following remarks are numbered to correspond with the numbers in parentheses in the text, and may be read (or not read) as footnotes to the text.

1. An additional circumstance essential in the introduction of the subject is that it is understood, tentatively, that such misalignments, expressed as distances or angles, are extremely small - of the order of hundredths of an inch and a fraction of a degree. This circumstance was largely responsible for the suggestion at hand, since it indicated at the outset that a first-order theory will suffice.

2. The basic hypothesis may not be universally accepted, since its proof is undoubtedly difficult, being in the nature of a successful completion of a ballistic analysis of rocket fire. The hypothesis is nevertheless very intriguing, as is any hypothesis that purports an explanation of large effects by small and difficult-to-measure causes; the hypothesis will remain an outstanding nuisance until definitely disproven. The suggested method of attack is radical. A nebulous heuristic analogy exists here with the subjects discussed in writer's BRLM 80, BRLM 685, BRLTN 353 and BRL 892.

3. If in a planar motion of a rigid body the directions of the velocities of two body-fixed points are known, the motion can be represented as a rotation about the point of intersection of the normals to those directions, erected at those points. In our case these normals are the arms themselves.

4. Let the length of the small side be 2D. The overhang of the large side then is $D(1 + 1/Q) - D = D/Q$, and the arms, therefore, make the angle $\gamma = \arctan(D/Qh)$ with the axis. Therefore the distance from the small side to the center of rotation is $D/\tan\gamma = hQ$.

5. The fact that in the first-order theory the path of the center of rotation is normal to the axis of the undisturbed trapezoid may be obvious intuitively. Yet the following relatively rigorous proof, incidental to the first-order theory of the idealized linkage, might be of interest.

6. We may note first, as a lemma, that both arms of the trapezoid move approximately through the same angle. Let one arm, whose length is $h\sec\gamma$, turn through a small angle ϕ; the displacement of the end of this arm is $\phi h\sec\gamma$, and is at the angle γ to undisturbed position of the large side. This displacement has a component $(\phi h\sec\gamma)\cos\gamma = \phi h$ along this side, and a normal component (in opposite directions for the two arms) of $(\phi h)\tan\gamma = \phi D/Q$. The relation between the angles through which the two arms turn is fixed by the fact that the distance between the two ends of these arms remains unchanged. But the normal components have only a second-order effect on that distance. Hence

the components along the long side must be the same, and hence the two angles are (to the first order) the same.

7. If both arms turn through the same angle, the angle between these two arms is unchanged. The point of intersection of the two arms then would travel in a circle passing through the two stationary pivot joints of the two arms; and to the first order, we are concerned only with a small portion of this circle, viz., a short line normal to the axis.

8. The representation of the motion as rolling without slipping is somewhat awkward in the linear theory: in this theory the path of the center of rotation with respect to the moving body, viz., in the coordinate system fixed in that body, or the curve which rolls (which we might call, for convenience, the polhode) is also a short segment of a straight line, i.e., it is indistinguishable from the path of the center of rotation in the fixed coordinates (viz., the curve on which the polhode rolls, which we might call the herpolhode). In the next approximation, if herpolhode is a circle passing through the vertices of the short side, the polhode is a circle passing through the vertices of the long side; the more exact curves may be readily constructed (or computed), and they indeed resemble somewhat these two circles. The representation of the motion as rolling, even though it is not an essential feature of the first-order theory, seems practically necessary in order to realize that the motion of the point fixed in the moving body at the original position of the center of rotation is only along the axis of the assembly, and is "of the second-order" with respect to the angle ϕ of the rotation of the arms: so that to the first order we may say that the displacement of that point is zero.

9. The parallelogrammic addition of forces determines not only the direction and the magnitude of the resultant, but also the line along which the resultant acts.

10. To prove the parallelism of the applied and the transmitted forces in the linear theory, consider the head immovable, and let the center of mass of the motor be in the original position of the center of rotation, as it is indeed the case for our practical purposes. Since the vector sum of the applied force and of the reaction of the head must be proportional to the acceleration of the center of mass of the motor, and since that acceleration in the first-order theory is zero, this vector sum must be zero, the reaction of the head must be antiparallel to the applied force (the jet thrust), or the transmitted force is parallel to the applied force. An extension of this reasoning by an inclusion of an axial acceleration of the rocket will present no difficulties. An attempt at a more exact analysis leads to considerable mathematical complications, which do not seem justified at this stage; particularly, since little use of this hypothetical parallelism is made in the text, and since the effect of these complications, as discussed in the text, should be expected to be small.

11. To evaluate the displacement of the center of rotation, consider the extensions of arms from the stationary pivots to the original position of the center of rotation. Their length is $hQ\sec\gamma$. As they turn through an angle ϕ, their ends move from this original position, at angles γ to the normal to the axis, through the distances $\phi hQ\sec^2\gamma$. Their new intersection is located, to the first-order theory, as discussed above, on that normal; and it is easy to see that this new intersection occurs at the distance $\phi hQ \sec^2\gamma$ from the original position. On the other hand, by considering either the displacements of the ends of the arms normal to the original position of the long side, $\phi D/Q$, in comparison with the half-length $D(1 + 1/Q)$ of that side, or the displacement $h\phi$ of the center of that side at the distance $hQ + h$ from the center of rotation, the angle through which the motor turns, θ, is $\phi/(Q+1)$. Thus both the displacement of the center of rotation and θ are proportional to ϕ; i.e., they are proportional to each other.

12. The intersection of the vector of the transmitted force with the axis of the head is distant from the center of rotation by $(\phi hQ\sec^2\gamma)\cot\theta = h(Q + Q^2)\ h\sec^2\gamma$. Subtracting hQ from this, the distance from the short side to this intersection is, $hQ^2(1 + (1 + 1/Q)\tan^2\gamma)$. While the angle γ in the first-order theory is not treated as an infinitesimal, it is easy to show that for a compact linkage (small h and large Q) the angle γ will be small, and the difference between the exact expression and the approximation hQ^2, unimportant - at least for the introductory purposes of the text.

13. An alternative term for the center of percussion of a trapezoidal linkage is the center of the transmitted force. Of course, this point exists only in the first-order theory. There exists some slight analogy: this point is conjugate, so to say, not to another point (center of rotation), but rather a (short) line (the tangent to the polhole and herpolole, just as the center of percussion of a rigid body is conjugate to a certain line (of the applied force). As the perusual of the text will show, no strong significance is attached to this point, and it is introduced purely for the sake of convenience of the presentation.

14. As mentioned above, the distance between the center of rotation and the center of percussion of the linkage is $h(Q+Q^2)\sec^2\gamma$, which approximately (for the purposes of presentation) is $h(Q+Q^2)$.

15. A body of mass M and radius of gyration K, acted upon by a force at a distance D from its center of mass, acquires a linear accleration (of its center of mass) of f/M, and an angular acceleration fD/MK^2; so that a point of this body lying on the intersection of f with the normal to f through the center of mass acquires an acceleration along f of $f/M + fD^2/MK^2 = (f/M)\ (1 + D^2/K^2)$. Hence the effective mass of the body at that point is $M/(1 + D^2/K^2)$, i.e., is less than M. The motion of the body, incidentally, is a rotation about its center of percussion, which is at the distance K^2/D from the center of mass, or at the distance $D + K^2/D$ from f.

16. A simple illustration might be helpful here. Let the masses and the radii of gyration of both head and motor be m and k, and let the distance between their centers of mass be $P = 2D$; consider the effect of the cross-force f at the center of mass of the motor.

(i) With conventional rocket the total mass is $M = 2m$ and the square of the radius of gyration of this mass is $K^2 = D^2 + k^2$. The angular acceleration is $fD/M(D^2 + k^2) = (f/MD)/(1 + k^2/D^2)$.

(ii) With our rocket, let $f = f_h + f_m$, these being the components acting upon the head and the motor, respectively. The acceleration of the center of mass of the motor is

$$a = f_m/m = f_h(1 + P^2/k^2)/m,$$

so that $f = ma + ma/(1 + P^2/k^2) = ma(2 + P^2/k^2)/(1 + P^2/k^2)$, whence

$$a = (f/m)\ (1 + P^2/k^2)/(2 + P^2/k^2)$$

The center of percussion of the head being at the distance $P + k^2/P$ from f, the angular acceleration of the head (i.e., of the center-line of our rocket) is

$$a/(P + k^2/P) = (f/m)\ (1 + P^2/k^2)/(2 + P^2/k^2)\ (P + k^2/P),$$

which can be written as

$$(f/MD)\ (1 + 4D^2/k^2)/(1 + 2D^2/k^2)\ (1 + k^2/4D^2)$$

(iii) Comparing (ii) with (i), we see that the ratio of the angular acceleration of the center-line of our rocket to the angular acceleration of the conventional rigid rocket is

$$(1 + 4D^2/k^2)/(1 + k^2/D^2)/\ (1 + 2D^2/k^2)(1 + k^2/4D^2),$$

which is always greater than 1.

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